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Structural Vulnerability and Excessive Public Indebtedness in CFA Franc Zone Countries

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Abstract

This paper relies on the ‘institutional debt rule’ implemented in Franc Zone countries to assess whether the structural vulnerability of these countries matter for their probability to enter into excessive indebtedness. This structural vulnerability is measured by retrospective ‘Economic Vulnerability Index’ (EVI) recently computed jointly by the United Nations and Guillaumont et al., (2011). We observe evidence that the impact of ‘EVI’ is non-linear with respect to the probability of these countries to engage into excessive indebtedness and that, this effect appears to be the same for the two monetary areas belonging to the CFA Franc Zone countries: a rise of EVI induces a higher probability of excessive debt and for higher EVI, this probability declines. Consequently, international development institutions such as the Bretton Woods should take into account such vulnerability in their assessment of the adequate development policies and recommendations to these countries.

Keywords: Structural Vulnerability; Public debt; unconditional logit model; linear probability model.

JEL Classification: E60; H63; O10 ; C33 ; C35.

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1. Introduction

The issue of conducting fiscal policy in a monetary union is longstanding. Fiscal discipline and fiscal restructuring in a monetary area have been the subjects of many theoretical and empirical studies in the developed world, particularly in the European Union. For example, many studies have been conducted on the determinants of excessive deficits in the euro area (e.g. Castro, 2007; Tiryaki, 2008; Bayar, 2001, 2009; Huges-Hallet and Lewis, 2004, 2005). However, to our knowledge, such topics have been scarcely explored in the context of African monetary unions such as the CFA¹ Franc Zone. This study aims to fill this gap.

The CFA Franc Zone was created in 1945 during the Bretton Woods agreement and it currently comprises 14 Sub-Saharan African countries that belong to two separate monetary areas: WAEMU (West African Economic and Monetary Union) and Economic Community of Central African States (ECCAS). These countries are also classified as the most vulnerable developing countries to natural and external shocks (see Guillaumont, 2009, 2011).

The present paper investigates whether this vulnerability of CFA Franc Zone countries matters for their public indebtedness. In other words, we rely on the budgetary institutional criteria (especially related to debt) set up in 1999 by WAEMU member countries and adopted by the end of 2001 by ECCAS members to explore whether the structural vulnerability of these countries matters for their excessive debt.

The remainder of the paper is organised in the following way. Section 2 presents a brief overview of the institutional arrangements of the CFA Franc Zone's monetary unions. Section 3 summarises the state of the literature on the definition and measurement of the concept of 'economic vulnerability' in order to derive our vulnerability index. Section 4 is devoted to some data analysis. Section 5 reviews the literature on debt sustainability, - since the setup of the institutional debt rule within the CFA Franc Zone aims at helping countries maintain a sustainable path of their public debt -, and sets the stage for the model used. Section 6, based on Section 5, describes the empirical model and discusses the expected signs of the variables and the econometrics technique. Section 7 presents the empirical results and Section 8 concludes.

¹ CFA was defined as 'Communauté Française d'Afrique', but is now known as 'Franc de la Communauté Financière d'Afrique' for WAEMU area and 'Coopération Financière en Afrique Centrale' for ECCAS area.

2. Institutional arrangements of CFA Franc Zone countries

Of the 14 Sub-Saharan African countries in the CFA Franc Zone, 12 were once French colonies. Formally, these countries belong to two separate monetary areas (WAEMU and ECCAS) and share two single currencies that hold the same acronym, the CFA Franc. Thus, the CFA Franc is issued and managed by two regional central banks: the Central Bank of West African States (known as *BCEAO* in French) and the Central Bank of Central African States (known as *BEAC* in French). These two CFA Francs were by design initially pegged at the same rate to the French Franc in 1948, and since 1999 to the Euro, following the creation of the euro area. Since the inception of the CFA Franc Zone, the French Treasury has guaranteed an unlimited convertibility of the CFA currencies and participated on the executive boards of the two regional central banks. The counterpart of this guarantee has been the obligation of each central bank to maintain a proportion of its official reserves (50% for *BCEAO* and 65% for *BEAC*) in an operation account at the French Treasury.

In line with the adoption by the European Union of the Maastricht treaty² in 1992, and recognising the crucial role of fiscal policy management in achieving macroeconomic stability, sustainable growth and macroeconomic convergence, both WAEMU and ECCAS have adopted a set of measures. In 1999, WAEMU member countries adopted a regional ‘Convergence, Stability, Growth and Solidarity Pact’, which defines a set of primary and secondary convergence criteria pertaining to public finance, the real sector, the balance of payments and common currency (the list of these criteria can be found in Adedeji and Williams, 2007 or the ‘Note d’information n°127 of Banque de France’³, 2010). ECCAS, following the establishment of a multilateral committee in 1993, adopted by the end of 2001 a framework of convergence criteria that comprises the same primary criteria as WAEMU. Accordingly, the two monetary areas share a set of primary criteria within the CFA Franc Zone. However, while a directive imposes sanctions against a WAEMU country’s non-compliance of a primary convergence criterion, such a sanction measure does not exist for ECCAS. The primary criteria include:

- The ratio of the basic fiscal balance to nominal GDP must be in balance or in surplus.

² This treaty comprises a set of rules reinforced in 1999 within the framework of the stability and growth pact for countries in the Economic and Monetary Union.

³ This note can be found at http://www.banque-france.fr/fileadmin/user_upload/banque_de_france/Information_diverses/infoetlib/note127.pdf.

- The ratio of outstanding domestic and foreign debt to nominal GDP must not exceed 70%.
- Average annual inflation rate cannot exceed 3% a year.
- The non-accumulation of domestic and external arrears.

The institutional debt rule is the core of our study. This rule, by constraining CFA Franc Zone countries to maintain their public debt under the threshold of 70%, acts as a debt sustainability rule. This is why Section 5 draws on the literature on debt sustainability to build the presented empirical model. Section 3 reviews the state of the literature on the definition and measurement of ‘economic vulnerability’.

3. The concept of ‘economic vulnerability’

3.1 A literature review on the definition of ‘economic vulnerability’

The concept of ‘vulnerability’ refers to that of ‘risk’. There are several definitions associated with the concept of ‘risk’ depending on the disciplines where it is studied. Generally, vulnerability can be seen as the risk that a ‘system’ undergoes from negative change due to a ‘perturbation’ (see e.g. Naudé et al., 2009).

In economics, vulnerability is either associated with poverty where the concern is the risk of households falling into or remaining in poverty, or natural hazards and macro-level shocks where the concern is how the hazards adversely affect a country or region’s economy (see e.g. Naudé et al., 2009). Guillaumont (2009) highlights that the first type of vulnerability can be derived from the second one. Our paper focuses on the second kind of vulnerability: the ‘economic or structural vulnerability’.

The issue of ‘economic vulnerability’ was really raised for the first time (in 1990) by the Maltese Ambassador, his Excellency Ambassador Alexander Borg Olivier⁴ (see Maltese Government, 1990: 7). Since then, many conceptual and empirical studies⁵ have been conducted on that issue. More specifically, Briguglio (e.g. 2004), Briguglio and Galea (2003), Cordina, (2004a, b) and Briguglio et al., (2008) define economic vulnerability as a country’s proneness to exogenous shocks lying outside their control or a proneness to increased susceptibility of such a country to the adverse effects of these shocks’.

⁴ He was the Permanent Representative of Malta to the United Nations.

⁵ The conceptual and empirical viewpoints of economic vulnerability are well documented in the literature (see e.g. Briguglio 1995, 2003; Atkins et al., 2000). Cordina and Farrugia (2005) also provide a summary on the measurement issue of the concept of ‘economic vulnerability’.

In the same vein, several studies of Patrick Guillaumont (see e.g. Guillaumont, 2009; Guillaumont and Cariolle, 2011) have been conducted on the issue of 'economic vulnerability' where he defines 'the economic or structural vulnerability of a country as the risk of a (poor) country seeing its development hampered by the natural and external shocks it faces'. Two main types of exogenous shocks (in other words, two main sources of vulnerability) are therefore considered:

- the environmental or 'natural' shocks which encompass, for instance, natural disasters (earthquakes, volcanic eruptions) and the more frequent climatic shocks (typhoons, hurricanes, droughts, floods, etc);
- external (trade-and-exchange-related) shocks which comprise, for instance, slumps in external demand, world commodity price instability (and correlated instability of terms of trade), international fluctuations of interest rates, and so forth.

Other domestic shocks such as unforeseen political changes are thus excluded from being exogenous.

Meanwhile, all these authors highlight the difference between the concept of 'economic vulnerability' and that of 'economic resilience'. For example, Briguglio (2008) defines the resilience as the policy-induced ability of an economy to recover from or adjust to the negative impact of adverse exogenous shocks and to benefit from positive shocks. Thus defined, economic resilience may take the form of higher savings and investments which may occur in the wake of pronounced uncertainty and may enable small island states to achieve high levels of economic development (Cordina, 2004). Guillaumont (2009) considers economic resilience as the capacity of a country to react to shocks. He underscores that this resilience depends more on current policy, is more easily reversed, and is less structural but may also comprise a structural element⁶.

Briguglio (2003) develops the notion of the 'Singapore Paradox', according to which many small island states, in spite of their economic vulnerability, manage to generate a relatively high GDP per capita when compared to other developing countries. To explain this phenomenon, Briguglio (2003, 2004) takes the case of Singapore which experiences high rates of economic growth and high GDP per capita despite its high exposure to external

⁶ According to Guillaumont (2009) a distinction close to this three components is given in Rodrik (1999) who, in looking at the risk of social conflict in countries facing external shocks, considers the individual severity of the shocks, the depth of latent social conflict (likely to increase the impact of the shocks), and the quality of conflict management institutions.

shocks. Hence, the ‘Singapore Paradox’ stems from the juxtaposition of economic vulnerability and economic (nurtured) resilience, where economic vulnerability was confined to inherent features which are permanent or quasi-permanent, while economic resilience was associated with man-made measures which enable a country to withstand or bounce back from the negative effects of external shocks.

3.2 A literature review on the measurement of economic vulnerability

In line with the definitions of economic vulnerability provided above, we summarise here the different measures of that concept. The propositions of vulnerability indices have mainly focused on the quantification of the special features of the countries by relying on indicators such as economic openness, export concentration, dependence on imports of energy and peripherality. Other approaches attempt to measure vulnerability in terms of the phenomenon, namely the variability of output and similar indicators.

The first vulnerability index was proposed by Briguglio (1993) and is composed of three variables: the exposure to foreign economic conditions, insularity and remoteness, and proneness to natural disasters. This index has been the subject of several modifications in 1995, 1997, and updated by Briguglio and Galea in 2003. Other authors such as Chander (1996) and Wells (1996) follow the methodology adopted by Briguglio (1995) and propose a vulnerability index which remains to a certain extent in line with Briguglio (1997)’s. Wells (1997) revised its measure of vulnerability and uses a methodology that departs from the previous ones by relying on the idea that ‘vulnerability manifested in instability in economic growth’. He then uses regression analysis to build its index. Atkins et al. (1998) also adopts the econometric analysis and show evidence that economic vulnerability captured by ‘output volatility’ depends mainly on the export dependency ratio, the merchandise export diversification and the vulnerability to natural disasters. Crowards (2000) also contributes to that literature by suggesting an index of economic vulnerability for developing countries which is similar to the previous ones, but is rather composed of more variables. In line with Wells’ (1997) study, the Committee for Development Policy (CDP)⁷ (2000) of the United Nations (UN) developed a composite index in order to identify the causes of vulnerability of least developed countries (LDCs). By capturing vulnerability through economic growth instability, this index is a weighted average of five variables, namely the share of manufacturing and modern services in GDP, merchandise export concentration ratio,

⁷ This committee was previously called Committee for Development Planning.

instability of agricultural production, instability of exports of goods and services and population size. The weights are obtained through an econometric analysis where the impact of each economic indicator quoted above on economic growth is examined. All these studies convey the same message according to which small states are inherently more vulnerable. However, Gonzales (2000) criticizes these studies, arguing that their results they lead to considerable variations and contradictions due to the differences of the parameters and the methodologies employed by them.

Following the renewal growing concern over macroeconomic vulnerability of least developed countries and the demand of these countries to build an adequate vulnerability indicator which should be taken into account in the design of international development policies, the CDP has developed and progressively refined, after successive revisions (2003, 2006 and 2009) an economic vulnerability index which captures vulnerability caused by structural factors. The structural economic vulnerability employed in this study referred to the so-called ‘retrospective Economic Vulnerability Index (EVI)’ jointly calculated on an annual basis by the FERDI⁸ (see Cariolle, 2011; Guillaumont and Cariolle, 2011) with the UN/United Nations Department of Economic and Social Affairs (DESA). This indicator covers 128 developing countries over the period 1975–2008 (unbalanced panel data) and has the advantage of being simple, transparent and parsimonious. Moreover, several multilateral development banks are exploring whether to move from their traditional indicator to EVI for aid allocation (see Guillaumont, 2011, for more details).

This ‘economic vulnerability’ is a result of three components: (i) the size and frequency of the exogenous shocks, either observed (ex post vulnerability) or anticipated (ex ante vulnerability); (ii) exposure to shocks; and (iii) the capacity to react to shocks, or resilience. Therefore, *structural vulnerability* (that is, the EVI), which results from factors that are independent of a country’s current political will is different from the *vulnerability deriving from policy*, which results from recent policy choices. In other words, an index of structural economic vulnerability is related to structural factors—not policy factors—that are beyond the present control of the country and which also influence global vulnerability, mainly through resilience (Guillaumont, 2009). This structural vulnerability index is a composite

⁸ FERDI is the ‘Fondation pour les Etudes et Recherches sur le Développement International’. The method of retrospective EVI’s calculation can be found in details in Cariolle (2011) and descriptive statistical analysis on the retrospective EVI can be found in Guillaumont (2011), and Guillaumont and Cariolle (2011). This is why we do not find it useful to replicate this statistical analysis here and refer the readers to those articles.

index of ‘shocks’ and ‘exposure to shocks’; both indicators are equally weighted⁹. We display below the structure of the retrospective EVI (henceforth, EVI) where the weights of indices are in brackets.

Structure of the EVI

{	Exposure Index (50%)	-Smallness (50%)
		-Location Index (Remoteness) (50%)
		-Specialization Index (Merchandise Export concentration and share of agriculture, forestry and fisheries) (25%)
{	Shock Index (50%)	-Natural Shock Index (Homelessness due to natural disasters; instability of agriculture production) (50%)
		-Trade Shock Index (Instability of exports of goods and services) (50%)

Source: Guillaumont et al. (2011)

4. Data analysis

Our study covers a sample of 14 CFA Franc Zone countries¹⁰ over the period 1980–2008. Within this group, eight countries belong to WAEMU¹¹ and six to ECCAS¹². Graph 1 in the Appendix compares the evolution of average total public debt with the average economic vulnerability index (EVI) for CFA Franc Zone countries. This graph suggests a strong correlation between average EVI and average total public debt over time.

Graph 2 illustrates the empirical distribution of the duration of ‘non-excessive debt’ spells, the latter being the time spent by a given CFA Franc Zone country within the state of ‘non-excessive debt’ (i.e., before entering ‘excessive debt’). This analysis provides an insight into the durations of ‘non-excessive debt’ spells for this monetary zone. We plot on the x-axis the observed spell lengths and on the y-axis the proportion of observations where the observed spell of non-excessive debt exceeds a given length. Note that of the total of 50 spells, there are 21 spells of ‘excessive debt’, or 42% of all spells.

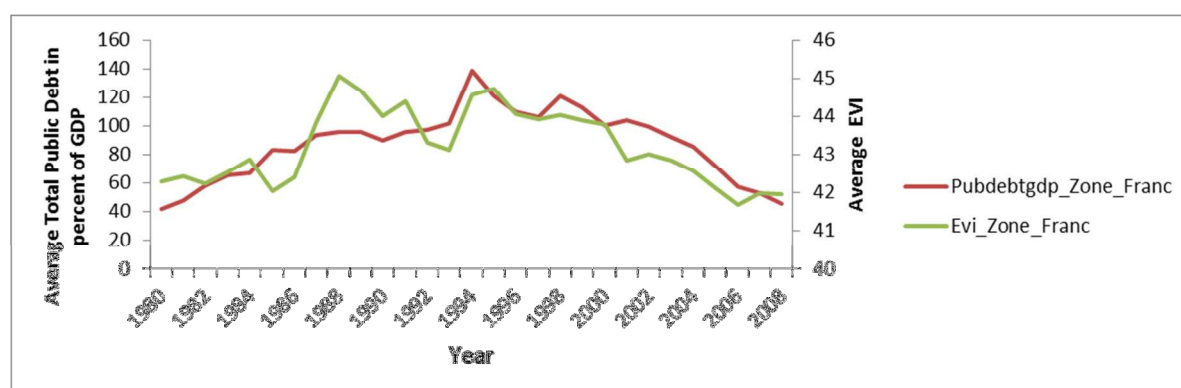
⁹ See for example Guillaumont and Cariolle (2011) for a discussion on the weight of indicators.

¹⁰ Benin, Burkina Faso, Cameroon, Central African Republic, Chad, Republic of Congo, Cote d’Ivoire, Equatorial Guinea, Gabon, Guinea-Bissau, Mali, Niger, Senegal and Togo.

¹¹ WAEMU countries include Benin, Burkina Faso, Cote d’Ivoire, Guinea-Bissau, Mali, Niger, Senegal and Togo.

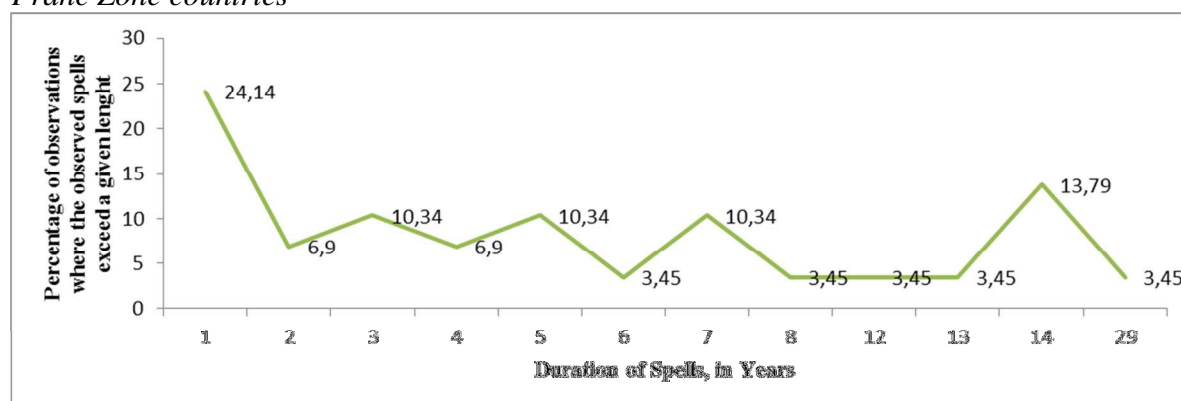
¹² ECCAS include Cameroon, Central African Republic, Chad, Republic of Congo, Equatorial Guinea and Gabon.

Graph 1: Comparative evolution of the Average Total Public debt and Average EVI of CFA Franc Zone Countries



Source: The Author Calculation based on IMF's data on Public Debt.

Graph 2: The empirical distribution of the Duration of 'Non-Excessive Debt Spells' in CFA Franc Zone countries



Source: The Author calculation is based on IMF's data on Public Debt. Figures in the graph represent the percentage of spells duration until the entry into excessive debt state in CFA Franc Zone where the observed spells exceed a given length.

Graph 2 also suggests for the CFA Franc Zone that among the spells of non-excessive debt and over the period 1980–2008, 24.14% enter into a state of “excessive debt” after the first year of non-excessive debt. After five years, more than half (58.6%) of spells enter into a state of ‘excessive debt’. The figure is approximately 76% after 8 years and approximately 96.55% after 14 years. Note that no spell lasts between 9 and 11 years and between 15 and 28 years. In addition, only one spell lasts 12 years, one, 13 years, four spells last 14 years and finally only one lasts 29 years. We can thus conclude that whereas a small proportion of spells of non-excessive debt’ are long-lasting, the most important ‘non-excessive debt’ spells last only a few years. Thus, CFA Franc Zone countries seem to display a high tendency to enter an ‘excessive debt’ state. Section 5 presents the traditional accounting mathematical model of the sustainability of public finances.

5. Sustainability of public finances

Since the purpose of our study is to examine the effect of economic vulnerability on the total public debt, we start with the standard public finances sustainability model and then derive the appropriate model that will help us perform our regressions. Although there is no consensus among economists regarding the public debt threshold that maintain the public debt path sustainable, the empirical literature distinguishes between three main approaches¹³ used to assess the public debt sustainability. These approaches have been discussed in IMF (2003) (see also Vera, 2009).

The first and most common approach starts from the basic accounting identity (or Domar's approach) and links the changes in the debt stock to public sector revenues and expenditures. According to this approach, fiscal policy is sustainable if it delivers a stable ratio of public debt to GDP. In other words, if the actual primary balance is less than the debt stabilizing balance, current fiscal policy that implies an increasing ratio of public debt to GDP is viewed as unsustainable. This approach allows calculating the so-called "debt stabilizing primary balance" which is the primary balance that would make the debt-to-GDP ratio stable. Hence, the degree of the needed fiscal adjustment stems from the difference between the actual and debt stabilizing primary balance. The second approach (a more flexible one) refers to the called Present-Value Constraint (PVC) approach. It assesses the debt sustainability within the context of the broader objectives and constraints of the fiscal policy decision-making process. For example, it consists in estimating fiscal policy reaction functions where the relationship between fiscal policy instruments and fiscal policy objectives (such as the stabilization of output fluctuations, the maintenance of debt sustainability) is examined. Hence, if the primary balance responds positively to public debt, this generally implies that fiscal policy is consistent with long-run solvency (see Bohn, 1998). The third approach to assessing public debt sustainability is to examine whether the government is "overborrowing", that is, its debt stock is higher than the present discounted value of its expected future primary surpluses. However, irrespective of the conceptual approach adopted, the fundamental block of the fiscal sustainability corresponds to a simplification of the government budget constraint (Vera, 2009).

¹³ Note that the World Bank and IMF have in the last few years defined the concept of public finance sustainability (or 'debt sustainability' according to their used expression) as in first time, a group of indicators and lately as a set of threshold (see IMF, 2002a).

By assuming that the government finances its deficit only by issuing debt (that is, other financing items such as seigniorage revenue, privatization proceeds, and the sales of public assets are excluded), this mathematical model can be written as follows (see Vera, 2009): $\Delta b_t = (i - g)b_{t-1} - (\tau - \phi)$ (1) where b_t denotes the ratio of the total public debt to GDP (Gross Domestic Product) and Δ represents the symbol of variation; i_t is the real interest rate; g is the real GDP growth rate; τ and ϕ are respectively the ratio of total tax revenue over GDP and the ratio of total government consumption (excluding interest payments on the total public debt) over GDP.

The equation (1) shows that if the primary surplus ratio is equal to zero, the debt-to-GDP ratio will grow or shrink at the rate $(i - g)$, within a framework where it is assumed that there is a level beyond which the debt-to-GDP ratio cannot or should not rise. Under this situation, unless there is a sufficient amount of primary budget surplus, the public debt ratio increases when the real effective interest rate on government debt exceeds the growth rate of GDP (that is, when the growth-adjusted real effective rate is positive). In other terms, the Domar's condition for debt stability (and thus fiscal sustainability) can be held when the real GDP growth rate is higher than the real interest rate, even if the primary balance continues to be just zero.

To estimate the effect of structural economic vulnerability on public debt in CFA Franc Zone countries, we rely in this paper on the mathematical model of fiscal sustainability underlying that fundamental block.

6. Model specification

In the previous model of fiscal sustainability, we assume that the budget deficit is financed only by debt creation. We now relax this hypothesis and consider the additional financing items that can add to government revenue (non-tax revenues) such as seigniorage revenue, privatization proceeds, and the sales of public assets. To take into account such items in the equation (1), we define the primary balance not as the difference between tax revenue and the government spending, but rather as the difference between the overall government revenue (excluding grants) - comprising several items (non-tax revenue) - and non interest (primary) expenditure.

Our model relies thus on the equation (1) augmented with our variables of interest (the EVI or its components), and other control variables which are likely to influence both the variables of interest and the dependent variable (the overall public debt).

We first present our model specification and discuss the expected signs of the covariates and finally the econometric technique.

6.1 The model specification

In this sub-section, we describe the model that allows us to examine the effect of the structural vulnerability of CFA Franc Zone countries in their probability of excessive indebtedness. More specifically, the model examines the probability of a country breaching the 70% of GDP debt rule (that is, leading to excessive public debt). The structural model is stipulated as follows:

$$\begin{cases} y_{it}^* = x_{it}'\beta + \varepsilon_{it} \\ y_{it} = 1 \text{ if } y_{it}^* > 70\%, \text{ and } y_{it} = 0 \text{ if } y_{it}^* \leq 70\% \end{cases}$$

where $i = 1, \dots, N = 14$, denotes the country index and $t = 1980-2008$, denotes the period (year) index; y^* is an unobserved outcome; y_{it} represents the excessive debt status: $y_{it} = 1$ if in a country i of the CFA Franc Zone at the year t , the government incurs an excessive public debt, that is, its total debt-to-GDP ratio is equal to or higher than 70%; $y_{it} = 0$, otherwise. The vector x_{it} represents the structural economic vulnerability variables (that is, the EVI or its components) as well as a set of other control variables which act as (economic) resilience-related variables which are supposed to influence the impact of EVI on the probability of excessive debt. These variables include the fiscal balance (in percentage of GDP), the real GDP growth rate, the terms of trade, the real effective exchange rate, the grants (as a percentage of GDP), the inflation rate (captured here through the GDP deflator), and a dummy variable representing the period since the entrance into force of the ‘‘Pact of Convergence, Stability, Growth and Solidarity’’ within the zone franc. The definition and the source of these variables are provided in the Appendix A. ε_{it} is an error term.

6.2 Discussion on the expected signs of the explanatory variables

Before starting the discussion of the expected effects (signs) of our explanatory variables, let us highlight one important pitfall regarding both the linear and the nonlinear model. Indeed, for the model’s parameters to be consistent and efficient, our regressors should be predetermined with respect to the dependent variable. In our case, to avoid any suspicion of endogeneity issue related to simultaneity bias between certain regressors and the dependent variable, we use in the model where it appears necessary the lagged values of the explanatory variables (especially for the variables ‘fiscal balance’, ‘real economic growth’, ‘inflation’,

grants-to-GDP ratio' and 'real effective exchange rate'). While this precaution could at least mitigate the simultaneity bias in any model like ours, in our specific case, the dependent binary variable is defined by institutional rules rather than by economic variables. As a result, there is likely no simultaneity bias and, the endogeneity issue will not thus be a problem. Nevertheless, to take into account such eventual problem along with the delay in processing some economic data, we do consider the previously quoted one year lagged variables in our different model specifications.

Now what about the expected effect of each explanatory variable?

The EVI's variable

In analyzing the effect of economic vulnerability on economic growth, Cordina (2004a, b) shows that increased risk can adversely affect economic growth as the negative effects of downside shocks would be commensurately larger than those of positive shocks. Furthermore, he presents a conceptual application of the "Singapore Paradox" approach and shows evidence that in response to a situation of vulnerability, saving and capital formation in an economy can be important sources of resilience. Guillaumont (2009) also discusses the effects of the (retrospective) EVI on economic growth and poverty and concludes that the EVI reduces economic growth and, through the latter, exerts deleterious effects on the pace of poverty reduction. These impacts occur through the channels of export earnings instability, the primary instabilities (especially through their effects on public finances or via the passed through of price fluctuations to producers), political instability, the smallness of the country, the structure of the economy and the location of the country.

More recently, Ferrarini (2009) re-assesses the analysis underlying the New Debt Sustainability Framework (NDSF) endorsed by both the Bretton Woods Institutions (the IMF and the World Bank) - which guides the borrowing decisions of Low-Income Countries. This re-evaluation consists in testing the significance and the reliability of the World Bank's CPIA or the governance indicators as predictors of debt distress episodes across LICs. He obtains strong evidence that factors of illiquidity and structural vulnerability¹⁴ are more suitable predictors of the occurrence of debt distress episodes across Low-Income Countries (LICs). Thus, by challenging the NDSF prospects whose aim is to solve the long-standing debt crisis

¹⁴ The Economic Vulnerability Indicator (EVI) used is that of United Nations Department of Economic and Social Affairs – Division of Sustainable Development.

involving many of the LICs, the author concludes that “the NSDF is bound to distort aid allocation away from the country-specific circumstances which truly matter for the achievement of debt sustainability”.

In our case, we argue the following: the structural vulnerability, by reducing tax revenue (unless the government increases taxes after an exogenous shock or when its exposures to shocks rises, though such measure is politically sensitive and difficult to implement) and increasing government spending is expected to increase the budget deficit. We hypothesise that, irrespective of its effect on the public finances (the budget balance), the structural vulnerability of a country is expected to increase its public indebtedness and thus to raise the probability of excessive indebtedness (especially in the case of CFA Franc Zone countries). However, one can think that when a government experiences a higher structural vulnerability of its economy, instead of borrowing even at low cost to cope with the additional financial needs induced by such rise in structural vulnerability, it can rely on non-costly financial means such as the privatisation proceeds, the seigniorage¹⁵ and the sale of its assets (buildings, infrastructure, mineral deposits, and various forms of liquid reserves) (Vera, 2009). In such cases an obtained positive effect of EVI on the accumulation of public debt or on the probability of excessive debt may cancel out the effect of the fiscal balance (the coefficient of the variable capturing the ‘the fiscal balance’ may not be statistically significant), since in the latter, we include tax revenue as well as other forms of non-tax revenues.

One can also think that countries facing structural vulnerability benefit from (temporary) debt forgiveness from their creditors, the extra revenues are used to cope with the shocks. In such situation, we can observe in the regressions a statistical significance of both the coefficient of the fiscal balance and that of the EVI.

We also conjecture that there exists a nonlinear (in the form of curve-linear or inverted U-shape) relationship between EVI/or its components and the probability of excessive debt and that, whether the membership of a CFA Franc Zone country to WAEMU or ECCAS matters for such non-linearity. The hypothesis underlying the inclusion of both the ‘EVI’/or its components and the square of EVI/or the square of its components is the following: When facing a rise in their vulnerability stemming either from exposure increases or shocks rises, or

¹⁵ Note however that only the ECCAS countries can use seigniorage to finance their deficits, as this measure is forbidden in WAEMU area.

both, a CFA Franc Zone country has several options apart from resorting to indebtedness to accommodate the additional costs induced by a such rise in structural vulnerability: the first option is to adjust its fiscal policies by either increasing taxes or reducing public spending or adopting both measures (taxes rise and expenditures reduction), although, as we mention above, these measures could be politically difficult to implement; the second option consists on using the non-costly means of financing (previously quoted) that are available to them. Note however that only the ECCAS countries can use seigniorage to finance their expenditures as this measure is forbidden in WAEMU area. The third option is for these countries to seek for debt relief granted by the creditors.

Therefore, we can have two main expectations regarding the nonlinearity of the EVI with respect to the probability of excessive debt in CFA Franc Zone countries:

- *Hypothesis 1*: we expect the public debt not to be affected or even to be reduced in the first stages of EVI increases (in the cases of fiscal adjustment or drawing on non-costly means of financing or debt relief), but as EVI becomes higher, these countries will have no choice but to resort to domestic and/or external debt. As a result, we expect a positive sign of the variable 'EVI square' (or the square of the components) and a negative sign of the coefficients associated to 'EVI' or its components. These coefficients may be statistically insignificant.
- *Hypothesis 2*: we also expect the likelihood of excessive public debt to rise in the first stages of EVI (if the countries choose to borrow internally or abroad to accommodate the EVI increases) and then to decline for higher EVI, because of the debt relief granted by creditors or the use of non-costly financial resources or also the adoption of fiscal adjustment measures. Hence, the sign of 'EVI' or its components will be positive and that of its square or its components' square will be negative.

The fiscal balance: as mentioned above, we expect an improvement in the fiscal balance to reduce the overall public debt and thus the probability of excessive debt. If the EVI (or its components) effect translates through the fiscal balance, the impact of the latter on public debt will be statistically nil. However, we acknowledge that a statistically nil effect of fiscal balance may not necessarily be due to the presence of the EVI or its components in the model, but may also be explained by the effect of other control variables of the model that influence the fiscal balance (e.g. economic growth; the terms of trade).

The real GDP growth: the indebtedness of a country is expected to rise following losses in output, that is, lowering of real GDP growth (see also Barro, 1979). Accordingly, the real

GDP growth is expected to be negatively associated with the build-up of public debt and accordingly, with the likelihood of excessive debt.

The real effective exchange rate

The real effective exchange rate (REER) indicates a country's competitiveness. In our case, a rise in the REER means an appreciation and a decline means depreciation. The effect of the real effective exchange rate on the overall public debt of a country depends on its effect on the domestic and external debt.

Regarding the domestic debt, on the supply side, its issuance may be easier to countries when the currency is appreciating because the expected appreciation allows prudent policymakers to hide the implicit insurance premium embedded in domestic currency borrowing (Caballero and Cowan, 2006; Panizza et al., 2011). On the demand side, a real appreciated exchange rate is, at any given interest rate, likely to discourage the demand of domestic currency bonds as investors may foresee an ex-post appreciation of the foreign currency rate (a real depreciation of local currency) (see also Panizza et al., 2011). Furthermore, in terms of valuation effects, a real effective exchange rate appreciation (depreciation) automatically induces a higher (lower) domestic indebtedness of the government.

With respect to the external public debt, the effect of REER changes on its build-up is also ambiguous. In fact, a real exchange rate depreciation will lead to a declining of the external debt stock if the induced rise in export earnings of this depreciation is sufficiently enough to service the external debt; otherwise, the depreciation of the REER will result in a rise of external indebtedness (Craigwell et al., 2010; see also Ng'eno, 2000), and hence the likelihood of excessive debt.

Overall, we cannot conclude a priori about the effect of REER changes on the probability of excessive debt.

The terms of trade

An improvement in terms of trade (an increase in the relative price of exportables for a country) is likely to increase the export (foreign) revenues of the beneficiary country, reduce current expenditures and therefore improve the fiscal balance. Note that the reduction effect of public expenditures owe to the terms of trade improvement appears through a relative decline in the price of inputs (in the cases where imports represent an important share of expenditures - which is usually observed in many developing countries and a fortiori, in CFA Franc Zone countries). Furthermore, such improvement in terms of trade, by increasing the

economic growth may also reduce the need for social assistance from government and in fine, add to the reduction of current expenditures. Thus, an improvement in terms of trade is expected to be positively related to lower external and /or domestic borrowing and by the same token, to a low probability of excessive debt.

Conversely, a decline in terms of trade, by lowering revenue, increasing (substantially) public spending and thus worsening the fiscal balance, will likely result in higher total public debt. As a result, the likelihood of excessive debt will rise. The positive effect of such terms of trade deterioration on public expenditures translated through for example, the rise of social assistance needs, and the high demand by public enterprises of support from the government because they cannot adjust their pricing policies to changes in export and import prices.

The grants

According to Cline (2003), in low-income countries (LICs), the grants elements (foreign grants, which represent a substantial fraction of GDP), are available to pay some part (or all) of the interest due on debt, and can consequently modify our previous debt sustainability. This is why we include in our model specification the foreign grants as a percentage of GDP. We thus expect the grants to alleviate the burden of indebtedness of developing countries—that is, to exert a negative effect on chance of entering into excessive debt. But we can also hypothesise that the higher the grants are for a developing country, the less it will be inclined to correctly manage its public finances to avoid unsustainable debt situations. In such instances, the grants will exert a positive effect on the total public debt.

The inflation rate

The impact of inflation on the public debt depends on how the latter is distributed among domestic and foreign creditors. In the case of developing countries and specifically in CFA Franc Zone countries, where usually a substantial part of the public debt is denominated in foreign currency, the inflation impacts directly the domestic debt-to-GDP ratio and indirectly the ratio of external debt to GDP through the real effective exchange rate.

A rise in inflation erodes the real value of the domestic debt held by creditors and the effective debt ratio, unless all domestic debt is indexed to prices or foreign currencies (though according to Panizza et al., (2011), in such cases inflation can debase indexed to prices if the government tinkers with the price index), a government can inflate away the domestic public debt by money creation, with the result of this inflating away of debt depending on the share of debt that is indexed to inflation. Although such policy measure can be implemented in

ECCAS, it can't in WAEMU (see above). Panizza et al. (2011)¹⁶ also point out the exceptional case where inflation can lead to a rise of public debt: in the case of a country facing a real appreciation (that is, where inflation outweighs the currency depreciation) and where a large share of domestic debt is indexed to inflation, the valuation effects will create a positive link between inflation and domestic currency debt.

6.3 Discussion on the choice of appropriate econometric technique

Since our dependent variable is binary (a dummy), we have to choose between two kinds of models: a linear probability model (LPM) and a non-LPM (logit or probit model). Whereas in LPMs, the probability of success and failure is considered to be a linear function of the covariates, in logit and probit models, the expected probability is an increasing non-linear function of the explanatory variables. However, compared with nonlinear models, there are several concerns regarding LPMs.

First, the marginal effect induced by a unit of variation of each covariate in an LPM is constant, whereas in nonlinear models it varies with each unit.

Second, in contrast to non-LPMs, the predicted probabilities of success or failure in an LPM may lie out of the interval [0,1]. On one side, Wooldridge (2002: 455) highlights that 'if the main purpose is to estimate the partial effect of the explanatory variables on the response probability, averaged across the distribution of these covariates, then the fact that some predicted values are outside the unit interval may not be very important. The linear probability model needs not provide very good estimates of partial effects at extreme values of the covariates'. In the same vein, Cameron and Trivedi (2005: 495) mention that ordinary least squares (OLS) estimations of such models provide a good guide to which variables are statistically significant. Ree and Nillesen (2009: 306–307) also emphasise that 'the probit/logit and LPM often produce rather similar outcomes because the conditional distribution function tends to 'look' rather linear around its expected value, while at the same time, most draws from any conditional distribution are 'close' to the expected value'. On the other side, Horrace and Oaxaca (2006) point out that OLS estimates of LPMs, where the predicted probabilities are outside the unit interval, may lead to biased and inconsistent estimates. They propose the sequential least squares (SLS) procedure as a way of remedying

¹⁶ These authors recognize however that such situation is exceptional and argue it will likely be dominated by the case where inflation impacts negatively the domestic debt, in the absence of financial repression.

this problem. These iterative procedure first trims from the data those OLS estimate observations with predictions lying outside the unit interval. Based on these estimations, the data are trimmed again and the model re-estimated. The procedure is repeated until no predictions are outside the unit interval and the SLS estimates are thus obtained.

Third, the problem of heteroskedasticity is likely to arise, leading to unbiased but inefficient coefficients (i.e., the standard errors are not valid for constructing confidence intervals and t-statistics). Weighted least squares are said to provide efficient estimates (Mullahy, 1990), but hold the disadvantage of having worse finite sample properties than OLS; further, the inferences based on asymptotic theory can be misleading (Alton and Segal, 1996)¹⁷. To overcome this difficulty, we use OLS with heteroskedasticity-robust standard errors (i.e., White's correction of heteroskedasticity).

Despite the above-mentioned drawbacks, LPMs have the particular advantage of facilitating the interpretation of the coefficients of interaction variables, whereas such an interpretation is not straightforward in logit or probit models. Indeed, Ai and Norton (2003)¹⁸ show that the marginal effect of an interaction term in nonlinear models, as provided by standard econometrics packages, may hold the wrong sign and significance and, consequently, cannot be interpreted as such. Greene (2010) challenges the way of interpreting Ai and Norton's (2003) results and notes that 'the process of statistical testing about partial effects, and interaction terms in particular, produces generally uninformative and sometimes contradictory and misleading results. The mechanical reliance on statistical measures of significance obscures the economic, numerical content of the estimated model' (p. 295). He recommends performing the analysis not only through statistical procedures (see Greene, 2010 for details) but also by graphical presentations. He also emphasises the need to take into account the units of measurement when interpreting the partial effects of continuous variables, as the partial effect (per unit change) can be misleading.

Kolasinski and Siegel (2010) also criticise Ai and Norton's (2003) interpretation of the interaction term. They first contend that in Ai and Norton's (2003) results 'it is difficult to interpret the sign of interaction term coefficient because for some observations, the cross

¹⁷ See also Wooldridge (2002: 455), who discourages the use of weighted least squares in LPM when the predicted probabilities lies outside the range [0,1].

¹⁸ According to these authors, the interaction effect is based on cross-partial derivatives with respect to the two interacted variables, which makes the sign and significance of the interaction term different for observations. They thus recommend relying on these derivatives and using the Delta method to assess the statistical significance of the marginal effect associated to the interaction term.

partial derivative of the probability of occurrence with respect to interacted covariates can have the sign opposite to that of the interaction term coefficient'. They argue that this is because of a mechanical saturation effect¹⁹, which is irrelevant for researchers primarily concerned with proportional marginal effects. For such researchers, small changes in probability are more important near the boundaries than they are near the centre. Kolasinski and Siegel (2010) conclude that the interaction term coefficient (provided by nonlinear logit or probit regressions) remains a valid measure of interaction because it is already purged of the saturation effect. Consequently, they suggest researchers who are not concerned with the saturation effect, use it as such, while others (those for whom the mechanical saturation effect is important) use Ai and Norton's (2003) measure of interaction.

In the case of only one interaction effect, researchers may use the easy fixes provided by standard econometrics packages (e.g., the use of 'inteff' ado-files in Stata) or the Delta method to obtain the interaction term in nonlinear models. However, these fixes become unusable in the case of double interaction effects ('inteff3' ado-files are available in Stata for the interaction of three dummies, but not for dummy(ies) and continuous variable(s)). Moreover, even using the Delta method to calculate standard errors, the computation becomes burdensome (as in our case here) with many interactions, especially with covariates²⁰ having high-order terms.

For all these reasons and given the ongoing discussion on the best way of obtaining good interaction terms and interpreting them in nonlinear models, we rely in this study on nonlinear models to perform our analysis and only use LPMs for interpreting interaction terms. In other words, the nonlinear model estimations allow us to interpret solely the non-interacted variables, while the LPM estimation allows us to interpret the coefficients of the interaction variables.

However, we still have three concerns. The first concern relates to the choice of fixed or random effects to model unobserved heterogeneity, the second focuses on which nonlinear

¹⁹ Kolasinski and Siegel (2010) explain why saturation effects might not be economically relevant in certain contexts. They particularly show that, under general conditions, the saturation effect guarantees that the Ai and Norton measure of interaction will have the opposite sign from the interaction term coefficient, as one or more of the covariates take on extreme values.

²⁰ In our case, these interaction variables include the square of this covariate, the interaction of this square with a dummy variable and the interaction of this covariate with another explanatory continuous variable of the model.

model (logit or probit) is suitable for our analysis and the third relates to the handling of temporal duration dependence.

- First, we use fixed effects, which capture heterogeneity among countries as well as the likely importance of unobservables correlated with the error term in determining the probability of excessive public debt, rather than random effects for two main reasons. First, since our sample is composed of heterogeneous countries, state-invariant and unmeasured factors (e.g., political, financial institutions, the degree of creditworthiness, etc.) are likely to be correlated with the error term in determining the evolution of the public debt-to-GDP ratio. Second, our macro panel contains the whole population of CFA Franc Zone countries (i.e., WAEMU and ECCAS countries) rather than a random sample from a much larger universe of countries where the use of random effects may be more suitable.
- Second, estimating standard dummy variables in a fixed-effects logit model²¹ using the unconditional Maximum Likelihood method can pose the incidental parameters problem, which presents significant challenges for obtaining unbiased parameter estimations. According to Neyman and Scott (1948), who first raised the issue of incidental parameters, the inconsistency of incidental parameters (fixed effects) arises because the number of incidental parameters N increases without bounds while the amount of information about each parameter is fixed. Hence, estimating a nonlinear model (especially an unconditional fixed-effects logit model) in large but narrow panels (with T fixed and N , the number of groups, growing infinitely) using the maximum likelihood method leads to severe bias (inconsistency) in the fixed effects and in the coefficients of the other control variables. This bias in the parameters is of the order of $1/T$ for balanced panels (Greene, 2011) and thus disappears as T becomes large. However, it is unclear which exact order of T produces unbiased estimates. Katz (2001) judges the performances of unconditional and conditional maximum likelihood estimators in fixed-effects logit models based on finite-sample properties where N units have been observed for T time periods. He shows evidence through a series of Monte Carlo experiments that for the time periods $T \geq 16$, unconditional and conditional maximum likelihood lead to the same

²¹ Note, however, that unconditional fixed-effects probit models are biased. In addition, the conditional fixed-effects model is not suitable in our case because only countries that display some heterogeneity in the outcome variable are taken into account in estimating the model. The requirement is binding in this setup given the small size of the cross-section dimension of our panel and the fact that for certain countries, the dependent variable takes either 1 or 0.

results. Further, Greene (2011, p. 621) illustrates in Monte Carlo simulations using a sample of $N = 1000$ with 200 replications that the bias of parameters is only about 6.9% when $T = 20$, but as large as 100% if $T = 2$. In this paper, our nonlinear model is estimated by relying on the unconditional fixed-effects logit model and, given the above discussion, the incidental parameter is not a problem in our case, as the temporal dimension of our panel is $T = 29$ years.

- Third, a concern when dealing with binary time-series cross-section models is how to model the temporal dependence (Beck et al., 1998), since in such situations, ordinary logit or probit models may result in too many inferences (too high t-statistics). The empirical literature offers several approaches to deal with the temporal dependence issue in such models: temporal dummy variables for each episode or ‘spell’ or specific transformations of time (duration variable) as covariates in the model (for e.g., a ‘linear’ time variable). Beck et al. (1998) show evidence that panel logit data are identical to grouped duration data and suggest dealing with this problem by adding a series of dummy variables to the model. These dummies should capture the number of years since the previous occurrence of an ‘event’. However, according to them, this solution has the drawback that it leads to an important loss of degrees of freedom (owing to the large number of dummy variables) and makes the hazard rate function likely to zig zag over time. Consequently, Beck et al. (1998) propose replacing the dummy variables with a smooth function based on ‘natural cubic splines’. This vector of spline-based variables, which are cubic polynomials of the time (t), smooth out the coefficients and the hazard function based on them. Hence, the number of spline variables will be lower than the number of time dummies; further, the statistical significance will be easier to achieve and the time dependence of the hazard function straightforward to test (see also the application of this technique in the recent paper by Castro and Martins, 2012). In this paper, we model the temporal dependence by using the ‘natural cubic splines’ as proposed by Beck et al. (1998). Moreover, we also follow another of Beck et al.’s (1998) suggestions by adding a variable that picks up the number of past events (e.g., the number of past episodes of non-excessive debt in our case). The inclusion of such a variable is justified by the fact that standard logit models assume ‘excessive debt’ states to be independent of one another, an argument which is obviously not true.

7. Empirical results

This section presents the results obtained from the statistical analysis (Table 1) and those obtained from the estimations of the different model specifications discussed above (Tables 2 to 5). Note that we standardise all our continuous covariates to allow meaningful economic interpretations.

Before presenting these results, we first highlight one shortcoming of this study. Although it is possible to estimate the different specifications of the model for the panel of WAEMU countries, such estimations are not possible for ECCAS sub-sample of countries either by the use of the logit model (the results do not converge) or by the use of the trimmed sample for the LPM (because in trimming the data, we are left with few observations, which prohibits statistical inferences). This shortcoming of the cross-sectional dimension of our panel prevents us from performing a graphical analysis of the interaction terms (as recommended by Greene, 2010). Further, this is especially the case when our variables of interest (EVI and its square/or EVI components and their squares) are interacted with the dummy WAEMU to measure how the partial effect of EVI – and its square/or EVI components and their squares – on the probability of excessive debt varies with a switch from WAEMU to ECCAS.

Table 1 shows the results obtained from performing a fractional polynomial analysis (FPA) to find out the correct parametric form for our variable of interest (the ‘EVI’ predictor), namely to check the linearity of that variable with the logit model (e.g., Hosmer and Lemeshow, 1999, 2000). The FPA consists of choosing between competing models by performing an Analysis of Deviance²² where the deviance statistics of these models are compared. The difference in the deviance between the two models is the likelihood ratio, G^2 , which has a null Chi-Squared distribution (Agresti, 2002). The FPA’s results suggest that the minimum deviance statistic (i.e., 123.21) is observed for model M2. In other words, model M2 is the best fitting nonlinear transformation that contains the ‘EVI’ and ‘EVI square’ variables. In fact, when comparing model M2 (the nonlinear model) with the linear model M1, we find evidence that the likelihood ratio test statistic, G^2 (i.e., the deviance for model M1 minus that for M2), is 7.638, with a significance level as follows: $p\text{-value} = \Pr(\chi^2(1) \geq 7.638) = 0.0057$. Since this p-value is lower than 0.01, we conclude that model M2 is significantly different from model M1 and therefore retains the nonlinear specification that contains the ‘EVI’ and ‘EVI square’ variables.

²² Note that the Analysis of Deviance is like the ANOVA used in multivariate normal regressions.

The inclusion of EVI's components in the analysis (see Tables 3, 5 and 6) allows us to explore whether the effects of 'EVI' on the chance of CFA Franc Zone countries entering into excessive debt are driven mainly by the variables 'exposure' and/or 'shock'. Nevertheless, we recognise that the effects of shocks on the probability of the excessive debt of a given country pertaining to this Zone could depend on the exposure of this country to shocks. To take into account (at least partially) the interaction between 'exposure' and 'shock', we include in the specification the variable 'exposureshock', which is the result of multiplying the 'exposure' and 'shock' variables (see the details in Appendix A).

Tables 2 to 5 have the same structures. We report in Columns [1], [2], [3] and [4] the results associated respectively to the unconditional fixed-effects logit model, the average partial effects regarding the latter, the results obtained from the LPM based on the whole sample and the results obtained from the LPM based on the trimmed sample (following the SLS procedure). Irrespective of the table considered (Tables 2 to 4), we find evidence that the results (sign, significance and magnitude of coefficients) of the LPM based on the whole sample are similar to the average partial effects of the unconditional logit specification, whereas the results of the LPM based on the SLS procedure are not²³. In addition, the magnitudes of the coefficients of the two LPMs are unexplainable differences.

Table 2 presents the results of the model specification comprising both the variables 'EVI' and its square. Following Greene (2010), we performed a simple Wald test of the zero interaction effect²⁴ for the coefficient of 'EVI' and that of 'EVI square' (see the Appendix D for the derivations of the logit model to obtain the interaction term, especially when applied to the simple case of a model specification where we have both a continuous variable and its square). As this test is sufficient but not necessary (Greene, 2010), it provides us with a good insight into the statistical significance of the 'EVI square' coefficient.

The result of this test shows that the interaction term (the coefficient of 'EVI' square) is statistically significant ($\chi^2(2) = 7.76$ with a p-value = 0.0206). Despite the divergence of the results between the LPM based on the whole sample and that based on the SLS procedure (Columns [3] and [4]), we obtain evidence that EVI is associated with nonlinearity with

²³ However, the sign and significance of certain estimates in the LPM based on the SLS procedure are identical to those of the logit model.

²⁴ As mentioned above, according to Ai and Norton (2003), the interaction effect is based on cross-partial derivatives with respect to the two interacted variables, which makes the sign and significance of the interaction term different for these observations.

respect to the likelihood of excessive debt in CFA Franc Zone countries. We can conclude that irrespective of the potential control variables, 'EVI' displays an inverted U-shaped curve relationship with the probability of entering into excessive debt in the CFA Franc Zone: an increase in one standard deviation of 'EVI' leads to a higher probability of excessive debt in this Zone; for higher levels of 'EVI', this probability decreases. The top point (turning point) of the 'standardised EVI' is approximately 0.50 for the LPM based on the whole sample and 0.4749 for the LPM based on the trimmed sample. As can be observed from these results, the turning points are almost the same. These results can be interpreted as follows: when the 'EVI' of these countries increases over time, they draw on their non-costly financial resources (see the details above) to cope with the additional financing needs induced by such a rise in structural vulnerability. However, after a certain level of 'EVI', these countries have no choice but to resort to either domestic or external debt, thereby increasing their likelihood of indebtedness. Hypothesis 2 thus seems to be valid here. Consequently, the same reasoning will be applied in cases where we observe a negative effect of 'EVI square' and a positive effect of 'EVI'.

In terms of the control variables, as shown by the Joint F-test on the duration dependence variables (Column [1]), there is a negative duration dependence in the behaviour of CFA Franc Zone countries compared with their likelihood of entering into excessive debt. In addition, the 'Convergence, Stability, Growth and Solidarity Pact' has lowered the likelihood of these countries entering into excessive debt compared with the period where such budgetary discipline does not exist (1980–1998). The probability of excessive debt is also positively driven by a rise in the grants, a rise in inflation, a depreciation in the real exchange rate and a deterioration in terms of trade. Real GDP growth and the fiscal balance seem to exert no statistical effect on the probability of excessive debt in CFA Franc Zone countries.

Table 3 contains the results of the model specification where 'EVI' and its square are replaced by EVI's components as well as their squares. The average partial effects (Column [2]) of the control covariates are roughly the same as those of Table 1 (Column [2]), except that here, the fiscal balance variable is positively associated with the probability of excessive debt in CFA Franc Zone countries. In other words, an improvement in the fiscal balance leads these countries to enter into excessive debt.

Concerning our variable of interest ('EVI's components and their squares), the results of the LPM based on the full sample are suggestive of a statistical significance (at the 1% level) of the variables 'shock', 'shock square' and 'exposure*shock'. The results of the LPM based on the SLS procedure suggest, however, that among our variables of interest, only 'shock square'

and ‘exposure square’ are statistically significant, although at the 10% significance level for the latter. Overall, we conclude from Table 3’s results that the nonlinearity observed for EVI with respect to the probability of excessive debt in CFA Franc Zone countries seems to be driven to a certain extent by both the ‘exposure’ and the ‘shock’ components of the ‘EVI’ variable.

Table 4 discriminates between the impact of EVI on the probability of excessive debt in WAEMU versus ECCAS. Once again, the average partial effects associated with the control explanatory variables and shown in Column [2] display roughly the same sign, magnitude and significance as those in Table 2. In addition, we observe as in Table 3 that an improvement in the fiscal balance increases the chance of CFA Franc Zone countries entering into excessive debt: a one standard deviation rise in the fiscal balance (i.e., 9.25%) increases the probability of excessive debt by 7.4% (although the statistical significance is only 10%). Moreover, irrespective of the model specification considered (logistic, linear probability based on the full sample or on the trimmed sample), WAEMU countries have a greater chance of entering into excessive debt than ECCAS countries. The result in Column [2] shows that holding all other covariates fixed, WAEMU countries have a 51.52% higher probability of entering into excessive debt compared with their ECCAS counterparts. Despite this difference in terms of behaviour, the inverted U-shaped relationship previously observed for EVI with respect to the probability of excessive indebtedness seems to be the same in terms of magnitude for WAEMU and ECCAS.

In Table 5, we use the model specification for which the results are displayed in Table 4 but replace the ‘EVI’ variable and its square by its components and their squares in addition to the interaction between the components. The average partial effects obtained for the control covariates are roughly the same as those reported in Table 4, except for the variable ‘dummyWAEMU’, where the average partial effect is higher than that in Table 4, and for the real exchange rate variable, which is here negative and significantly related to the likelihood of excessive indebtedness of CFA Franc Zone countries. As stipulated above, we cannot interpret the interaction terms associated with our variables of interest based on the average partial effects reported in Column [2]. Evidence is shown from Column [3] of Table 5 that despite the previously observed absence of a difference between WAEMU and ECCAS countries in terms of the nonlinearity of EVI with respect to the probability of excessive debt, a simultaneous rise in both ‘exposure to shocks’ and ‘shocks’ seems to exert a higher impact in WAEMU than in ECCAS countries. Further, the coefficient of the variable ‘shock square’ interacted with the variable ‘dummyWAEMU’ is negative and significant. Since the other

interaction variables with the ‘dummyWAEMU’ are not statistically significant at the 10% level, the interpretation of such results would be based on the combination of the two previous results. Hence, from the results of the LPM based on the full sample, we find that a one standard deviation increase in ‘exposure’ and a one standard deviation rise in ‘shock’ will lead to a probability of excessive debt = $(-1.184 + 1.59) = 0.406 = 40.6\%$ higher in WAEMU than in ECCAS countries.

We highlighted above the first shortcoming of our study. Another shortcoming is the difficulties in using a political/institutional variable. In fact, we intend to see whether the quality of governance in these countries alleviates or even renders statistically nil the effect of ‘EVI’ (and its square) on the probability of excessive debt and whether once taking into account this variable, the effects of our variables of interest differ between WAEMU and ECCAS countries.

For this reason, we need to introduce the variable ‘quality of governance’ into the model. However, since the data on this variable are not available for many countries of our sample and given the small size of the latter, we cannot use either the logistic model or the LPM based on the trimmed sample to perform this analysis. Accordingly, to have an idea of such effect, we rely solely on the LPM based on the full sample (the data available are also not sufficient to apply the SLS procedure), the results of which are presented in Table 6. The idea underlying the introduction of an institutional variable in the analysis is that the better the quality of institutions, the lower the build-up of public debt and thus the lower the likelihood of entering into excessive indebtedness. In addition, there is a need for developing countries that are structurally vulnerable to set up adequate institutions to promote competitiveness, build economic resilience and promote sustainable development (Farrugia, 2007). Thus, institutions in developing countries, particularly in CFA Franc Zone countries, should be as strong as possible to reflect the governance aspects inside their economic environments.

By assuming that the variable ‘quality of governance’ really captures the quality of governance in these countries, the results in Table 6 show that although this variable appears with the expected sign, it is not statistically significant. Moreover, it does not affect the nonlinearity relationship between EVI and the probability of excessive debt in CFA Franc Zone countries. The magnitude associated with this nonlinear relationship is the same for WAEMU and ECCAS countries, even if the probability of excessive debt itself is higher in WAEMU countries than it is in ECCAS countries. The results obtained for the EVI’s components and their squares, as well as their interactions with the ‘dummyWAEMU’ variable, are consistent with the estimates in Table 5. With regard to the other control

variables, the likelihood of CFA Franc Zone countries entering into excessive debt seems to be driven positively by a depreciation in the real exchange rate or deterioration in terms of trade. The ‘Convergence, Stability, Growth and Solidarity Pact’ has exerted a negative impact on this probability, while the duration dependence remains negative and significant. The remaining control variables do not seem to be statistically significant. We conclude that if the variable ‘quality of governance’ really captures the quality of governance in CFA Franc Zone countries, an improvement in such quality in these countries does not affect the relationship observed between EVI and the probability of these countries entering into excessive indebtedness.

8. Conclusion and policy implications

By using the ‘debt rule’ among other criteria implemented by both WAEMU and ECCAS countries within the CFA Franc Zone for macroeconomic stability and convergence purposes, this paper assesses whether the structural vulnerability of such countries affects their indebtedness, and more particularly their likelihood of entering into excessive debt. To perform our analysis, we use the (structural) EVI jointly computed by Guillaumont et al. (2011) and the UN-DESA (United Nations –Department of Economic and Social Affairs) and focus on a panel of 14 CFA Franc Zone countries over the period 1980–2008. We also replace in the model specifications the ‘EVI’ variable with its components. Overall, the results are suggestive of a nonlinear effect of ‘EVI’ with respect to the probability of entering into excessive debt: for a rise in EVI, the probability of excessive debt increases in CFA Franc Zone countries; however, for higher EVI, this probability significantly declines.

The policy implications of these results is that international development institutions such as the World Bank and IMF should take into account such vulnerability in their assessment of the adequate development policies and recommendations - especially those related to debt issues -, to these countries.

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Appendix, tables and Graphs

Table 1: Fractional Polynomial Analysis

Model	Hypothesis to be tested	df	Deviance	G ²	P-Value
M0: The variable 'EVI' is not included in the model		0	131.56482		
M1: The variable 'EVI' is included in the model (Linear Model with respect to 'EVI')	M0 nested in M1	1	130.85196	0.712856	0.3985
M2: 'EVI' and its square are included in the model (Nonlinear Model with respect to 'EVI')	M1 nested in M2	1	123.21382	7.638146	0.0057

Note: df = Degree of Freedom; G² = Likelihood Ratio associated to the difference of Deviances; P-Value = Probability associated to G².

Table 2: EVI's Effect on the probability of Excessive Debt in CFA Franc Zone Countries

Explanatory Variables	Dependent Variable : Excessive Debt Dummy			
	Logit Model (Fixed Effects)	Average marginal effect (partial effects) ^a	Linear Probability Model on the whole sample	Linear Probability Model on the trimmed sample ^b (SLS procedure)
	(1)	(2)	(3)	(4)
Evi	17.80*** (6.444)	1.037467*** (.3544797)	1.149*** (0.370)	3.702*** (1.231)
Evisq	-18.46*** (6.890)	-1.076052*** (.3793959)	-1.150*** (0.397)	-3.897*** (1.265)
Fiscal_balance _{t-1}	1.024 (0.703)	.0596867 (.0405361)	0.0255 (0.0340)	0.204 (0.157)
Gdpgrowth _{t-1}	-0.328 (0.341)	-.019126 (.0196781)	-0.0360* (0.0214)	-0.0232 (0.0694)
Inflation _{t-1}	-0.770* (0.416)	-.044911* (.0238273)	-0.0410 (0.0265)	-0.135 (0.0938)
Grantsgdp _{t-1}	1.802*** (0.564)	.1050366*** (.0304828)	0.0573 (0.0385)	0.796*** (0.276)
REER _{t-1}	-0.767* (0.447)	-.0447252* (.0255027)	-0.121*** (0.0308)	-0.143 (0.0925)
Termstrade	-3.058*** (0.742)	-.1782468*** (.0392632)	-0.126*** (0.0295)	-0.501*** (0.167)
Pacte	-3.229*** (1.022)	-.1882356*** (.0561457)	-0.143*** (0.0479)	-0.444** (0.203)
Number	-2.308*** (0.557)	-.1345159*** (.0276998)	-0.217*** (0.0402)	-0.488*** (0.118)
Variable	-1.095*** (0.228)	-.0638374*** (.0107247)	-0.0657*** (0.0123)	-0.139* (0.0743)
Spline (1)	-0.00107 (0.00119)	-.0000624 (.0000687)	-0.000317** (0.000147)	0.000443 (0.00157)
Spline (2)	-0.000170 (0.00233)	-9.91e-06 (.0001358)	0.000234 (0.000160)	0.0257** (0.0113)
Spline (3)	-0.0122*** (0.00423)	-.0007086*** (.0002316)	-0.000282 (0.000336)	-0.0192* (0.0103)
Observations	317		344	99
Log likelihood	-61.606908			
Wald Chi2 ^c / F-Statistic (P-Value)	64.44 (0.0000)		157.20 (0.0000)	8.63 (0.0000)
R Squared			0.838	0.7546
Joint F-test on « Duration dependence » variables	23.27 (0.0001)		8.40 (0.0000)	5.18 (0.0010)
Fixed Effects Significance	51.41 (0.0000)		141.24 (0.0000)	12.95 (0.0000)

Note: *p-value<0.1; **p-value<0.05; ***p-value<0.01. All these models contain country-dummies fixed effects. Standard Errors are in parenthesis. a: The average marginal effects are computed using the Delta method. b: This is the Sequential Least Square procedure of Horrace and Oaxaca (2006). c: The Wald test and the P-Value associated concerns the logit model where all coefficients are tested to be jointly equal to zero. The F-statistic and the P-value are associated to the linear probability models.

Table 3: *EVI's components Effect on the probability of Excessive Debt in CFA Franc Zone Countries*

Explanatory Variables	Dependent Variable : Excessive Debt Dummy			
	Logit Model (Fixed Effects)	Average marginal effect (partial effects) ^a	Linear Probability Model on the whole sample	Linear Probability Model on the trimmed sample ^b (SLS procedure)
	(1)	(2)	(3)	(4)
Exposure	-9.258 (9.356)	-.4834815 (.486234)	-0.357 (0.467)	-2.849 (1.827)
Shock	12.79 (7.775)	.6676987* (.396062)	1.220*** (0.332)	1.881 (1.466)
Exposuresq	16.83* (9.780)	.878965* (.5013442)	0.837 (0.518)	3.900* (2.082)
Shocksq	-9.284*** (3.060)	-.4848228*** (.147198)	-0.460*** (0.177)	-2.064*** (0.579)
Exposureshock	-4.495 (9.033)	-.2347203 (.4702039)	-1.005*** (0.335)	-0.0285 (1.596)
Fiscal_balance _{t-1}	1.977** (0.873)	.1032258*** (.0440164)	0.0437 (0.0341)	0.635*** (0.196)
Gdpgrowth _{t-1}	-0.297 (0.381)	-.0154988 (.0197703)	-0.0342 (0.0212)	-0.0470 (0.0806)
Inflation _{t-1}	-0.896* (0.480)	-.0467909* (.0244584)	-0.0404 (0.0250)	-0.174 (0.121)
Grantsgdp _{t-1}	1.913*** (0.619)	.0998922*** (.0301187)	0.0371 (0.0374)	1.074*** (0.275)
REER _{t-1}	-0.893* (0.463)	-.0466562** (.0232502)	-0.131*** (0.0323)	-0.132 (0.0828)
Termstrade	-3.205*** (0.814)	-.1673953*** (.0377766)	-0.122*** (0.0297)	-0.461*** (0.154)
Pacte	-2.965*** (1.051)	-.1548515 (.051094)	-0.162*** (0.0539)	-0.477** (0.180)
Number	-3.180*** (0.724)	-.1660735*** (.0310941)	-0.227*** (0.0409)	-0.847*** (0.147)
Variable	-1.263*** (0.261)	-.0659435*** (.0109898)	-0.0652*** (0.0121)	-0.153** (0.0754)
Spline (1)	-0.00161 (0.00125)	-.000084 (.0000646)	-0.000336** (0.000151)	0.000845 (0.00349)
Spline (2)	0.000521 (0.00281)	.0000272 (.0001465)	0.000293* (0.000165)	0.0320*** (0.0109)
Spline (3)	-0.0152*** (0.00489)	-.0007923*** (.0002358)	-0.000345 (0.000351)	-0.0245* (0.0130)
Observations	317		344	83
Log likelihood	-55.237813			
Wald Chi2 ^c / F-Statistic (P-Value)	58.24 (0.0010)		146.62 (0.0000)	6.75 (0.0000)
R Squared			0.840	0.784
Joint F-test on « Duration dependence » variables	23.46 (0.0001)		8.63 (0.0000)	6.66 (0.0002)
Fixed Effects Significance	38.82 (0.0001)		127.51 (0.0000)	9.38 (0.0000)

Note: *p-value<0.1; **p-value<0.05; ***p-value<0.01. All these models contain country-dummies fixed effects. Standard Errors are in parenthesis. a: The average marginal effects are computed using the Delta method. b: This is the Sequential Least Square procedure of Horrace and Oaxaca (2006). c: The Wald test and the P-Value associated concerns the logit model where all coefficients are tested to be jointly equal to zero. The F-statistic and the P-value are associated to the linear probability models.

Table 4: 'EVI's Effect on the probability of Excessive Debt in WAEMU versus ECCAS

Explanatory Variables	Dependent Variable : Excessive Debt Dummy			
	Logit Model (Fixed Effects)	Average marginal effect (partial effects) ^a	Linear Probability Model on the whole sample	Linear Probability Model on the trimmed sample ^b (SLS procedure)
	(1)	(2)	(3)	(4)
Evi	1.737 (12.73)	.0985157 (.7219109)	0.289 (0.495)	0.574 (1.888)
Evisq	-1.007 (14.75)	-.0571219 (.8364606)	-0.255 (0.484)	-0.516 (2.091)
dummyWAEMU	3.631*** (1.202)	.2059909*** (.0622984)	0.890*** (0.162)	2.753*** (0.452)
Eviwaemu	22.56 (14.56)	1.279856 (.8120269)	2.279*** (0.810)	3.971* (2.183)
Evisqwaemu	-24.28 (16.88)	-1.377162 (.9428261)	-2.470*** (0.858)	-4.232* (2.437)
Fiscal balance _{t-1}	1.307* (0.790)	.0741478* (.0441364)	0.0649** (0.0327)	0.218 (0.196)
Gdpgrowth _{t-1}	-0.196 (0.372)	-.0111365 (.0210011)	-0.0386* (0.0215)	-0.0734 (0.0732)
Inflation _{t-1}	-0.718* (0.431)	-.0407349* (.0240257)	-0.0374 (0.0266)	-0.0792 (0.0821)
Grantsgdp _{t-1}	1.735*** (0.575)	.0984442*** (.030467)	0.0496 (0.0377)	0.309* (0.162)
REER _{t-1}	-0.733 (0.464)	-.0416072 (.02585)	-0.118*** (0.0297)	-0.100 (0.0980)
Termstrade	-3.089*** (0.781)	-.1752252*** (.0405236)	-0.146*** (0.0281)	-0.455*** (0.170)
Pacte	-3.137*** (1.040)	-.1779594*** (.0557497)	-0.110** (0.0488)	-0.533** (0.243)
Number	-2.448*** (0.594)	-.1388575*** (.028832)	-0.259*** (0.0399)	-0.411*** (0.112)
Variable	-1.086*** (0.228)	-.0615867*** (.0105242)	-0.0652*** (0.0120)	-0.190*** (0.0514)
Spline (1)	-0.00104 (0.00124)	-.0000593 (.0000703)	-0.000336** (0.000150)	-1.29e-05 (0.000204)
Spline (2)	-0.000590 (0.00248)	-.0000335 (.0001404)	0.000182 (0.000165)	-0.000298 (0.00100)
Spline (3)	-0.0112*** (0.00432)	-.0006339*** (.0002327)	-0.000164 (0.000337)	-0.00221* (0.00128)
Observations	317		344	111
Log likelihood	-60.179742			
Wald Chi2 ^c / F-Statistic (P-Value)	64.22 (0.0001)		157.14 (0.0000)	8.82 (0.0000)
R Squared			0.843	0.749
Joint F-test on « Duration dependence » variables	22.86 (0.0001)		8.50 (0.0000)	3.71 (0.0079)
Fixed Effects Significance	31.64 (0.0009)		132.71 (0.0000)	10.75 (0.0000)

Note: *p-value<0.1; **p-value<0.05; ***p-value<0.01. All these models contain country-dummies fixed effects. Standard Errors are in parenthesis. a: The average marginal effects are computed using the Delta method. b: This is the Sequential Least Square procedure of Horrace and Oaxaca (2006). c: The Wald test and the P-Value associated concerns the logit model where all coefficients are tested to be jointly equal to zero. The F-statistic and the P-value are associated to the linear probability models.

Table 5: 'EVI's components Effect on the probability of Excessive Debt in WAEMU versus ECCAS

Explanatory Variables	Dependent Variable : Excessive Debt Dummy			
	Logit Model (Fixed Effects)	Average marginal effect (partial effects) ^a	Linear Probability Model on the whole sample	Linear Probability Model on the trimmed sample ^b (SLS procedure)
	(1)	(2)	(3)	(4)
Exposure	-17.26 (18.12)	-.8111736 (.8479976)	-1.656** (0.671)	-4.004 (2.887)
Shock	18.51 (13.15)	.8698677 (.6040299)	1.176*** (0.370)	5.433* (3.079)
dummyWAEMU	10.96*** (2.796)	.5151719*** (.1117504)	1.523*** (0.189)	2.349*** (0.764)
Exposuresq	27.86 (17.70)	1.309631 (.8181585)	1.944*** (0.641)	7.009** (3.240)
Shocksq	2.916 (5.787)	.1370845 (.2716366)	-0.0337 (0.160)	0.851 (0.983)
Exposureshock	-27.43* (15.27)	-1.28945* (.6934194)	-1.522*** (0.341)	-8.453** (3.767)
Exposurewaemu	-2.192 (22.34)	-.1030191 (1.050018)	0.964 (1.114)	0.825 (3.947)
Shockwaemu	-4.572 (15.96)	-.2148894 (.7488446)	0.0277 (0.690)	-3.176 (3.933)
Exposuresqwaemu	1.554 (23.00)	.0730482 (1.080952)	-0.892 (1.276)	-1.754 (4.270)
Shocksqwaemu	-14.99** (6.886)	-.7046872** (.3102294)	-1.184*** (0.301)	-3.134** (1.320)
Exposureshockwaemu	27.45 (18.62)	1.290239 (.8557206)	1.588* (0.904)	8.937* (4.959)
Fiscal_balance _{t-1}	2.094** (0.987)	.0984357** (.0450082)	0.0556* (0.0320)	0.527* (0.287)
Gdpgrowth _{t-1}	-0.239 (0.452)	-.0112262 (.0211115)	-0.0436** (0.0178)	-0.0309 (0.137)
Inflation _{t-1}	-1.034** (0.519)	-.0486139** (.0234128)	-0.0407* (0.0239)	-0.201 (0.135)
Grantsgdp _{t-1}	1.951*** (0.633)	.0917232*** (.0272569)	0.0553 (0.0382)	0.503** (0.212)
REER _{t-1}	-1.464** (0.631)	-.0688191** (.0284533)	-0.119*** (0.0284)	-0.325* (0.179)
Termstrade	-3.165*** (0.947)	-.1487908*** (.0411214)	-0.169*** (0.0293)	-0.661** (0.306)
Pacte	-2.821** (1.121)	-.1326018*** (.0499856)	-0.144*** (0.0513)	-0.493* (0.273)
Number	-3.316*** (0.723)	-.1558461*** (.0260734)	-0.302*** (0.0412)	-0.713*** (0.178)
Variable	-1.334*** (0.291)	-.0626965*** (.0110908)	-0.0648*** (0.0117)	-0.347*** (0.113)
Spline (1)	-0.00190 (0.00153)	-.0000892 (.000071)	-0.000279* (0.000144)	-0.000273 (0.000248)
Spline (2)	0.00242 (0.00423)	.0001136 (.0001987)	0.000285* (0.000163)	0.00149** (0.000734)
Spline (3)	-0.0172*** (0.00620)	-.000807*** (.0002734)	-0.000408 (0.000334)	-0.00565*** (0.00193)
Observations	317		344	80
Log likelihood	-49.682483			
Wald Chi2 ^c / F-Statistic (P-Value)	53.34 (0.0185)		136.23 (0.0000)	5.07 (0.0001)
R Squared			0.860	0.737
Joint F-test on « Duration dependence » variables	21.34 (0.0003)		8.92 (0.0000)	3.30 (0.0183)
Fixed Effects Significance	23.72		45.32 (0.0000)	9.38 (0.0000)

Note: *p-value<0.1; **p-value<0.05; ***p-value<0.01. All these models contain country-dummies fixed effects. Standard Errors are in parenthesis. a: The average marginal effects are computed using the Delta method. b: This is the Sequential Least Square procedure of Horrace and Oaxaca (2006). c: The Wald test and the P-Value associated concerns the logit model where all coefficients are tested to be jointly equal to zero. The F-statistic and the P-value are associated to the linear probability models.

Table 6: 'EVI's components Effect on the probability of Excessive Debt in WAEMU versus ECCAS: Taking in account the 'Quality of Governance'

Explanatory Variables	Dependent Variable : Excessive Debt Dummy	
	Linear Probability Model on the whole sample	Linear Probability Model on the whole sample
	(1)	(2)
Evi	2.035** (0.888)	
Evisq	-2.059* (1.075)	
Eviwaemu	-1.020 (1.169)	
Evisqwaemu	1.194 (1.382)	
Exposure		0.0379 (1.032)
Shock		1.367* (0.707)
dummyWAEMU	0.739*** (0.133)	1.728*** (0.274)
Exposuresq		0.965 (1.172)
Shocksq		1.794** (0.887)
Exposureshock		-2.929*** (1.072)
Exposurewaemu		-0.538 (1.390)
Shockwaemu		-0.669 (0.853)
Exposuresqwaemu		-0.392 (1.592)
Shocksqwaemu		-2.968*** (0.999)
Exposureshockwaemu		3.717*** (1.205)
Qog	-0.143 (0.243)	-0.350 (0.284)
Fiscal_balance _{t-1}	-0.0238 (0.0417)	-0.0157 (0.0374)
Gdpgrowth _{t-1}	-0.0234 (0.0300)	-0.0318 (0.0277)
Inflation _{t-1}	-0.00488 (0.0261)	-0.00169 (0.0256)
Grantsgdp _{t-1}	0.0133 (0.0283)	0.0143 (0.0250)
REER _{t-1}	-0.117*** (0.0387)	-0.108** (0.0452)
Termstrade	-0.147*** (0.0346)	-0.138*** (0.0354)
Pacte	-0.100* (0.0600)	-0.189*** (0.0659)
Number	-0.374*** (0.0532)	-0.335*** (0.0574)
Variable	-0.102*** (0.0207)	-0.102*** (0.0236)
Spline (1)	-0.000519*** (0.000175)	-0.000435** (0.000213)
Spline (2)	0.000251* (0.000135)	0.000268 (0.000164)
Spline (3)	-0.000148 (0.000260)	-0.000316 (0.000331)
Observations	218	218
F-statistic (Pvalue)	291.48 (0.0000)	246.25 (0.0000)
R Squared	0.921	0.931
Joint F-test on « Duration dependence »	11.94 (0.0000)	14.53 (0.0000)
Fixed Effects Significance	9.72 (0.0000)	12.79 (0.0000)

Note: *p-value<0.1; **p-value<0.05; ***p-value<0.01. All these models contain country-dummies fixed effects. Standard Errors are in parenthesis. a: The average marginal effects are computed using the Delta method. b: This is the Sequential Least Square procedure of Horrace and Oaxaca (2006). c: The Wald test and the P-Value associated concerns the logit model where all coefficients are tested to be jointly equal to zero. The F-statistic and the P-value are associated to the linear probability models.

Appendix A: Definition and Source of variables

Variable and Definition	Source and Comments
Dummyexcessivedebt	The author 's computation is based on the IMF's database on Gross Public Debt – The IMF's database weblink on Gross Public debt is: http://www.imf.org/external/ns/cs.aspx?id=262 . This dummy variable takes the value '1' if the total Gross Public Debt-to-GDP ratio is higher than 70% and the value '0', otherwise.
EVI = Economic Vulnerability Index	Guillaumont and Cariolle, J. (2011).
Exposure = Exposure Index	We use the 'exposure index' provided by Guillaumont and Cariolle, J. (2011) that we divide by 2, as the 'EVI' is the average of Exposure Index and Shock Index.
Shock = Shock Index	We use the 'exposure index' provided by Guillaumont and Cariolle, J. (2011) that we divide by 2, as the 'EVI' is the average of Exposure Index and Shock Index.
Fiscal_balance = Fiscal Balance in percentage of GDP	Centre d'Etudes et de Recherches sur le Developpement International (CERDI)'s Public Finance Database. Fiscal balance is the overall revenue (tax and non-tax revenue), excluding grants minus government expenditures.
Gdpgrowth = Real Gross Domestic Product (GDP) growth (annual %)	World Development Indicators (WDI) 2011
Termstrade = Net barter terms of trade index (2000 = 100)	World Development Indicators (WDI) 2011
REER = Real Effective Exchange Rate, Base 100 = 2000	CERDI's Database: This is the Real Effective Exchange Rate, base 2005 = 100 computed by CERDI: it is the ratio of prices in the country to prices in the main import partners adjusted for variations in nominal effective exchange rate. An increase means an appreciation.
Grantsgdp = Grants in percent of GDP.	Grants data are grants disbursements by all donors expressed in current millions of US Dollars. They are extracted from the OECD Statistical Database. The GDP used to calculate the ratio of Grants in percentage of GDP is extracted from the World Development Indicators (WDI) 2011.
Inflation = Inflation, GDP deflator (annual %)	World Development Indicators (WDI) 2011
DummyWAEMU	This an indicator variable taking the value '1' if the country of CFA Franc Zone is WAEMU's member, and '0', otherwise.
Number	This variable indicates the number of prior excessive debt episodes (or spells)
Pacte	This is a dummy variable capturing the entry into force of the 'Stability, Growth and Solidarity Pact' within the zone franc since 1999. It takes the value '1' for the years 1999-2008, and '0', otherwise.
Quality of Governance	The quality of governance is measured by subjective indices from the International Country Risk Guide (ICRG). The quality-of-governance index from ICRG used here is an 18-point scale, created by summing the following three six-point scales: corruption in government, bureaucratic quality, and the rule of law. See the ICRG for the criteria used in coding these measures. The rationale for corruption and bureaucratic quality is obvious. The rule-of-law definition indicates that this measure reflects the government's administrative capacity in enforcing the law, as well as the potential for rent-seeking associated with weak legal systems and insecure property rights. Source: International Country Risk Guide (ICRG) Data.
<i>Other Variables</i>	
Evisq = The square of 'EVI'; Exposuresq = the square of 'Exposure'; Shocksq = the square of 'shock'; Exposureshock = Exposure*Shock; Eviwaemu = Evi*DummyWAEMU; Evisqwaemu = Evisq*DummyWAEMU; Exposuresqwaemu = Exposuresq*DummyWAEMU; Shocksqwaemu = Shocksq*DummyWAEMU; Exposureshockwaemu = Exposureshock*DummyWAEMU.	

Appendix B: Statistics on Periods of 'Excessive debt' on CFA Franc Zone countries

Country	Period of the data on Total Public Debt availability	Period of 'Excessive debt'
Benin	1980-2008	1983; 1985; 1989-1991; 1993-1995
Burkina Faso	1980-2008	'No identified period of 'excessive debt'
Cameroon	1980-2008	1994-2003
Central African Rep.	1980-2008	1994-2008
Chad	1980-2008	1994; 1999-2000
Congo, Republic of	1980-2008	1981-2007
Côte d'Ivoire	1980-2008	1982-2008
Equatorial Guinea	1985-2008	1985-1996
Gabon	1980-2008	1987-1989; 1994-2003
Guinea-Bissau	1986-2008	1986-2008
Mali	1980-2008	1982-2001
Niger	1980-2008	1994-2002
Senegal	1980-2008	1983-1988; 1994-1999; 2001
Togo	1980-2008	1980-1989; 1991; 1993-2007

Appendix C: Descriptive Statistics

Variable	Observations	Mean	Standard Deviation	Minimum	Maximum
Dummyexcessivedebt	393	0.524173	0.5000519	0	1
Evi	401	43.22752	9.917783	24.67814	72.40889
Exposure	406	26.19997	5.148306	16.6997	40.30086
Shock	401	17.0898	6.899618	5.937748	37.36527
Fiscal_balance	375	4.34111	9.248568	-17.94905	38.85204
Gdpgrowth	400	3.705248	7.968115	-28.09998	71.18799
Inflation	400	7.950738	15.72924	-29.17266	112.8948
Grantsgdp	401	9.980065	9.037769	0.193719	49.40295
REER	406	126.1705	34.87212	40.2845	259.0221
Termstrade	401	123.2465	55.78856	21.27743	357.5757
Pacte	406	0.2413793	0.4284478	0	1
Qog	245	0.3803713	0.1215567	0.1666667	0.6944445

Appendix D: Interaction Effects

We provide here a general derivation of interaction effects in both linear and non-linear models, following Ai and Norton (2003) and Ai, Norton and Wang (2004). In fact, these authors demonstrate on the basis of a model comprising two interacted variables that the interaction effect is based on cross-partial derivatives with respect to the two interacted variables, which makes the sign and significance of the interaction term different for observations.

This Appendix is structured as follows:

- Firstly, we present a general derivation of interaction effects in linear models.
- Secondly, we focus on non-linear models, specifically logit model, where we consider interaction effects only for variables that do not have high order terms (that is for example, a continuous variable and its square): the demonstration is performed for two different cases: the first one deals with two interacted continuous variables and the second deals with two interacted variables where one is continuous and the other is a dummy.
- Thirdly, and lastly, we consider the case of non-linear (logit) model which contains continuous variables with high order terms.

1- Interaction effects in linear models

Consider the following linear specification of the expected value of dependent variable:

$$E[y / x_1, x_2, x] = \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + x^T \beta \quad (1)$$

where β_1 , β_2 and β_{12} are parameters and β is a vector of parameters, all of them are unknown. x^T is a vector of variables that excludes x_1 and x_2 (that is, x^T is supposed to be independent of x_1 and x_2).

Assuming that x_1 and x_2 are continuous variables. The marginal effects of x_1 on the expected

$$\text{value } E = E[y / x_1, x_2, x] \text{ depends on } x_2 \text{ if } \beta_{12} \neq 0: \frac{\partial E}{\partial x_1} = \beta_1 + \beta_{12} x_2 \quad (2)$$

The interaction effect given by the impact of a marginal change in x_2 on the marginal effect

$$\text{of } x_1, \text{ is thus obtained from taking the derivative of (2) with respect to } x_2: \frac{\partial^2 E}{\partial x_2 \partial x_1} = \beta_{12} \quad (3).$$

As a consequence, in linear specifications, the interaction effect $\frac{\partial^2 E}{\partial x_2 \partial x_1}$ equals the marginal effect $\frac{\partial E}{\partial(x_2 x_1)} = \frac{\partial E}{\partial w} = \beta_{12}$ where w represents the interaction term. However, in non-linear models such as logit and probit, this inequality does not hold. In the next section, we provide a general derivation of interaction effects in logit model where the interaction variables are continuous and where there is no high order terms in the model.

2- Interaction effects in non-linear models: the case of logit model without high order terms

Consider now the following expected value of dependent variable y :

$$E[y | x_1, x_2, x] = F(\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + x^T \beta) = F(u) \quad (4)$$

Where $F(u)$ is a nonlinear function of its argument $u = \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + x^T \beta$.

For example, in the logit model, $F(u)$ equals the cumulative distribution function defined as

$$F(u) = \frac{\exp(u)}{1 + \exp(u)} \text{ where } \exp \text{ denotes the exponential function. For probit model, } F(u) \text{ would}$$

be the cumulative normal distribution function $\Phi(u)$. Since in this paper, we have used the logit model to perform our analysis, we will focus on this function for the derivation of the formulae of interaction effects if

- (i) x_1 and x_2 are both continuous variables ;
- (ii) x_1 is a continuous variable and x_2 is dummy variable.

(i) If $F(u)$ is a twice differentiable function, with the first and second derivatives denoted by $F'(u)$ and $F''(u)$ being respectively the marginal effect with respect to x_1 , we obtain the interaction effects formulae as following :

$$\frac{\partial^2 E}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_2} \left(\frac{\partial E}{\partial x_1} \right) = \frac{\partial}{\partial x_2} \left(\frac{\partial F(u)}{\partial x_1} \right) = \frac{\partial}{\partial x_2} [F'(u) \frac{\partial u}{\partial x_1}] = \frac{\partial}{\partial x_2} [f(u) \frac{\partial u}{\partial x_1}] \quad (5)$$

The transformation of (5) leads to:

$$\frac{\partial}{\partial x_2} [f(u) \cdot (\beta_1 + \beta_{12} x_2)] = f(u) \cdot \beta_{12} + (\beta_1 + \beta_{12} x_2) (\beta_2 + \beta_{12} x_1) f'(u) \quad (6)$$

For logit model where $F(u) = \frac{\exp(u)}{1 + \exp(u)}$, evidence is shown that $F'(u) = [F(u)][1 - F(u)]$ and

$$F''(u) = [F'(u)][1 - 2F(u)] = [F(u)][1 - F(u)][1 - 2F(u)].$$

As a result,

$$\frac{\partial^2 E}{\partial x_2 \partial x_1} = \beta_{12} \cdot [F(u)][1 - F(u)] + (\beta_1 + \beta_{12}x_2)(\beta_2 + \beta_{12}x_1)[F(u)][1 - F(u)][1 - 2F(u)] \quad (7)$$

This suggests that the interaction effect $\frac{\partial^2 E}{\partial x_2 \partial x_1}$ generally differs from the marginal effect

$$\frac{\partial^2 E}{\partial (x_2 x_1)}$$
 of the interaction term $w = x_2 x_1$.

(ii) Assume now that x_1 is a continuous variable and x_2 is dummy variable.

Consider the expected value function (4) where x_1 is a continuous variable and x_2 is dummy variable.

The mixed interaction effect $\frac{\Delta}{\Delta x_2} \left(\frac{\partial E}{\partial x_1} \right)$ can be computed as follows:

$$\frac{\Delta}{\Delta x_2} \left(\frac{\partial E}{\partial x_1} \right) = \frac{\Delta}{\Delta x_2} \left(\frac{\partial F(u)}{\partial x_1} \right) = \frac{\partial F(u)}{\partial x_1} \Big|_{x_2=1} - \frac{\partial F(u)}{\partial x_1} \Big|_{x_2=0} = 0 \quad (8)$$

The transformation of (8) leads to the following expression:

$$\frac{\Delta}{\Delta x_2} \left(\frac{\partial E}{\partial x_1} \right) = F'(\beta_1 x_1 + \beta_2 + \beta_{12} x_1 + x^T \beta) - \beta_1 F'(\beta_1 x_1 + x^T \beta) \quad (9)$$

Note however that $\frac{\Delta}{\Delta x_2} \left(\frac{\partial E}{\partial x_1} \right) \neq \frac{\Delta}{\Delta x_1} \left(\frac{\partial E}{\partial x_2} \right)$.

3- *Interaction effects in non-linear models: the case of logit model with one high order terms (order 2)*

(i) *Consider the case of logit model with one high order terms (order 2) – this the case for our model with the variable ‘EVI’ and ‘EVIsquare’*

Instead of expectation (4), we now depart from the following expected value:

$$E[y / x_1, x_2, x] = F(\alpha_1 x_1 + \beta_1 x_1^2 + x^T \beta) = F(u) \quad (10)$$

where $u = \alpha_1 x_1 + \beta_1 x_1^2 + x^T \beta$.

The formula of the interaction effect is:

$$\frac{\partial^2 E}{\partial x_1 \partial x_1} = \frac{\partial}{\partial x_1} \left(\frac{\partial E}{\partial x_1} \right) = \frac{\partial}{\partial x_1} \left(\frac{\partial F(u)}{\partial x_1} \right) = \frac{\partial}{\partial x_1} [F'(u) \frac{\partial u}{\partial x_1}] = \frac{\partial}{\partial x_1} [(\alpha_1 + 2\beta_1 x_1) \cdot F'(u)]$$

Hence, $\frac{\partial^2 E}{\partial x_1 \partial x_1} = 2\beta_1 F'(u) + (\alpha_1 + 2\beta_1 x_1) \cdot [F''(u)] \cdot \frac{\partial u}{\partial x_1}$ and finally,

$$\frac{\partial^2 E}{\partial x_1 \partial x_1} = 2\beta_1 [F(u)][1 - F(u)] + (\alpha_1 + 2\beta_1 x_1)^2 [F(u)][1 - F(u)][1 - 2F(u)] \quad (11)$$

(ii) *Consider the case of logit model with two high order terms (order 2) – this the case for our model which contains the components of the variable ‘EVI’ and their squares.*

Instead of expectation (10), we now depart from the following expected value:

$$E[y / x_1, x_2, x] = F(\alpha_1 x_1 + \alpha_2 x_2 + \beta_1 x_1^2 + \beta_2 x_2^2 + \beta_{12} x_1 x_2 + x^T \beta) = F(u) \quad (12)$$

where $u = \alpha_1 x_1 + \alpha_2 x_2 + \beta_1 x_1^2 + \beta_2 x_2^2 + \beta_{12} x_1 x_2 + x^T \beta$.

The interaction term of the variable x_1^2 is obtained through the following expression:

$$\frac{\partial^2 E}{\partial x_1 \partial x_1} = \frac{\partial}{\partial x_1} [F'(u) \frac{\partial u}{\partial x_1}] = \frac{\partial}{\partial x_1} [(\alpha_1 + 2\beta_1 x_1 + \beta_{12} x_2) \cdot F'(u)] = 2\beta_1 \cdot F'(u) + (\alpha_1 + 2\beta_1 x_1 + \beta_{12} x_2)^2 F''(u)$$

Therefore, $\frac{\partial^2 E}{\partial x_1 \partial x_1} = 2\beta_1 \cdot [F(u)][1 - F(u)] + (\alpha_1 + 2\beta_1 x_1 + \beta_{12} x_2)^2 [F(u)][1 - F(u)][1 - 2F(u)] \quad (13)$

Similarly, the interaction term of the variable x_2^2 is given by:

$$\frac{\partial^2 E}{\partial x_2 \partial x_2} = 2\beta_2 \cdot [F(u)][1 - F(u)] + (\alpha_2 + 2\beta_2 x_2 + \beta_{12} x_1)^2 [F(u)][1 - F(u)][1 - 2F(u)] \quad (14).$$

Overall, we conclude that for linear functions where $F(u) = u$ and where $F'(u) = 1$, the interaction effect is given by β_{12} , for the nonlinear function such as logit, it is always different from β_{12} , even if the model contains variables with high order terms.